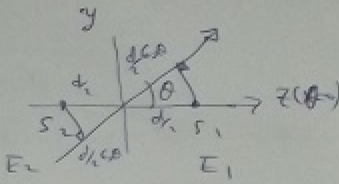
Sheet (5)
Solution

(1) given 2 isotropic, spaced $\lambda/2$, in phase, find D, E
identical = same Amplitude

sol/



$E_t = E_1 + E_2$

$E_1 = E_o e^{j\psi/2}$ → [illegible]

$E_2 = E_o e^{-j\psi/2}$

$E_t = E_o \left(e^{j\psi/2} + e^{-j\psi/2} \right) = 2E_o \cos(\psi/2)$

$\therefore E_n = \cos \psi/2$

where $\psi/2$ is phase between 2 sources $= \beta \frac{d}{2} \cos\theta$

$\therefore E_n = \cos\left(\frac{\beta d}{2} \cos\theta\right)$

at $\lambda/2$ $\quad E_n = \cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda/2}{2} \cos\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$

→ to draw Pattern (مش مطلوبة لنا)

E_n → max at E_{max} $\frac{\pi}{2}\cos\theta = \pm n\pi$ $\quad \therefore \cos\theta = \pm 2n$

at $n=0$ → $\cos\theta = 0$ $\quad \boxed{\theta_{max} = \pm 90^\circ}$

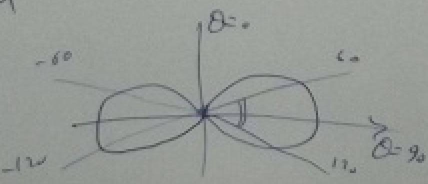
θ_{nulls} → at $E=0$ $\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$ or $\frac{\pi}{2}\cos\theta = \pm(2n+1)\pi/2$

$\cos\theta = \pm(2n+1)$ $\quad n=0$ → $\boxed{\theta_n = 0, 180}$

θ_{HP} → $\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\cos\theta = \pm(2n+1)\pi/4$ $\quad \therefore \cos\theta = \pm(n + 1/2)$

at $n=0$ θ_h $\boxed{\pm 60, \pm 120}$

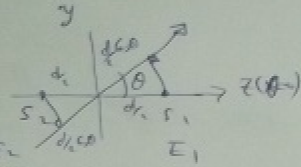


~~D~~ $= \theta_{HP} = 2|90-60| = 60^\circ$

Sheet (5)
Solution

① given 2 isotropic, spaced $\lambda/2$, find D, E
identical = same Amplitude, in phase

sol/



$E_t = E_1 + E_2$

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→ to draw Pattern ([illegible])

$E_n \longrightarrow$ max at E_n $\frac{\pi}{2}\cos\theta = \pm n\pi$ $\quad \therefore \cos\theta = \pm 2n$

at $n=0 \longrightarrow \cos\theta = 0$ $\quad \boxed{\theta_{max} = \pm 90^\circ}$

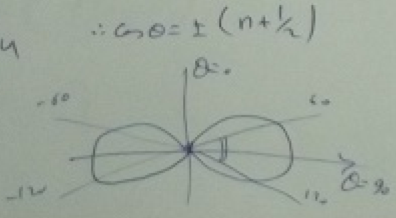
$\theta_{nulls} \longrightarrow$ at $\underline{\underline{E=}}\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$ $\quad$ or $\frac{\pi}{2}\cos\theta = \pm(2n+1)\pi/2$

$\cos\theta = \pm(2n+1)$ $\quad n=0 \longrightarrow \boxed{\theta_n = 0, 180}$

$\theta_{HP} \longrightarrow \cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\cos\theta = \pm(2n+1)\pi/4$ $\quad \therefore \cos\theta = \pm(n+\frac{1}{2})$

at $n=0$ only θ_h $\boxed{\pm 60, \pm 120}$



~~Bapprox~~ $= \theta_{HP} = 2|90-60| = 60^\circ$

②

$$D_{exact} = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \cos^2(\frac{\pi}{2}\cos\theta) \cdot \sin\theta \, d\theta \, d\phi}$$

$\Omega_A \rightarrow \iint E_n^2 \sin\theta \, d\theta \, d\phi$

$$D = \frac{\cancel{4\pi} \; 2}{\cancel{2\pi} \int_0^{\pi} \cos^2(\frac{\pi}{2}\cos\theta) \cdot \sin\theta \, d\theta}$$

نعوضها بالتكامل عشان نكامل

let $x = \frac{\pi}{2}\cos\theta$

$dx = -\frac{\pi}{2}\sin\theta \, d\theta$

$\int_{\theta=0}^{\pi} \longrightarrow \int_{x=\pi/2}^{-\pi/2}$

$$\therefore D = \frac{2}{\int_{\pi/2}^{-\pi/2} \cos^2 x \cdot \left(-\frac{2}{\pi}dx\right)}$$

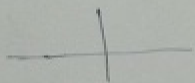
$$= \frac{-\pi}{\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_{\pi/2}^{-\pi/2}} = \frac{-\pi}{\left[-\frac{\pi}{4} + 0 - \frac{\pi}{4} + 0\right]} = 2$$

<u>Problem (2)</u>

2 isotropic, identical, $d = \lambda/2$, <u>out of phase</u> (phase opposition)

find E_n

sol



$$E = E_0 e^{j\psi/2} - E_0 e^{-j\psi/2}$$

out of phase

[illegible] $E = E_0 e^{j\psi/2} + E_0 e^{-j(\psi/2 + \pi)} = E_0 e^{j\psi/2} - E_0 e^{j\psi/2}$

$$E = 2j E_0 \sin(\psi/2)$$

$$E_n = \sin(\psi/2) = \sin\left(\frac{\beta d \cos\theta}{2}\right)$$

for $\lambda/2$ $\quad E_n = \sin\left(\frac{\pi}{2}\cos\theta\right)$

③

قاله ا D تقسم الطرق الى ستة هنوصل

برضه لازن $D = 2$

كولها مشابه

$$\int \sin^2 x \, dx$$

$$= \frac{x}{2} - \frac{\sin 2x}{4}$$

Pattern

max $\sin\left(\frac{\pi}{2}\cos\theta\right) \rightarrow \pm 1$

$$\frac{\pi}{2}\cos\theta = \pm(2n+1)\,\pi/2$$

$$\cos\theta = \pm(2n+1)$$

$$n=0 \rightarrow \theta_{max} = 0, 180$$

nulls $\sin\left(\frac{\pi}{2}\cos\theta\right) = 0$

$$\frac{\pi}{2}\cos\theta = \pm n\pi$$

$$\cos\theta = \pm 2n$$

$$n=0 \rightarrow \cos\theta = 0 \qquad \pm 90 \rightarrow \theta_n$$

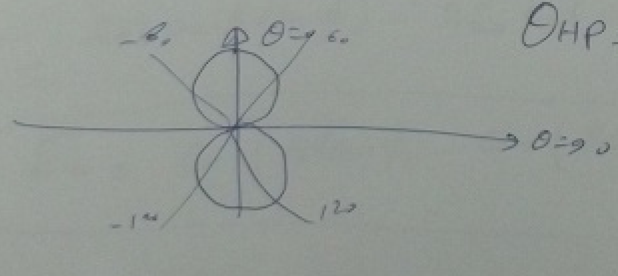
HP $\longrightarrow$ $\sin\left(\pi/2 \cos\theta\right) = \pm \frac{1}{\sqrt{2}}$

$$\pi/2 \cos\theta = \pm(2n+1)\,\pi/4$$

$$\cos\theta = \pm\left(n + \tfrac{1}{2}\right)$$

$$n=0 \rightarrow \pm 60, \pm 120$$

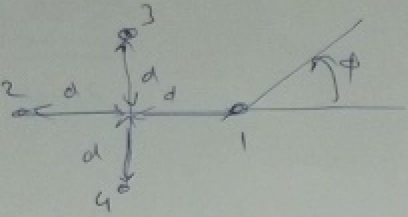
~~n=1 x~~



$$\theta_{HP} = 2|0 - 60| = 120^\circ$$

4

Problem (3)



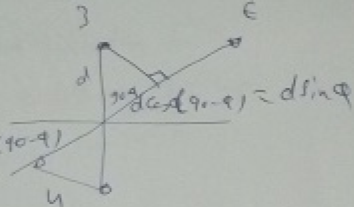
if $d = 3\lambda/8$

$S_1, S_2 \Rightarrow$ in phase

$S_3, S_4 \rightarrow$ opposite phase with 1, 2

find E

sol

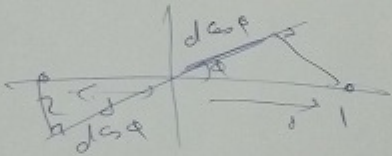


S_3, S_4

$E_3 = E_0 e^{j\beta d\sin\phi}$

$E_4 = E_0 e^{-j\beta d\sin\phi}$

$E_{3,4} = E_0 e^{j\beta d\sin\phi} + E_0 e^{-j\beta d\sin\phi} = 2E_0\cos(\beta d\sin\phi)$



S_1, S_2

$E_1 = E_0 e^{j\beta d\cos\phi}$

$E_2 = E_0 e^{-j\beta d\cos\phi}$

$\therefore E_{1,2} = E_0 e^{j\beta d\cos\phi} + E_0 e^{-j\beta d\cos\phi} = 2E_0\cos(\beta d\cos\phi)$

out of phase

$\therefore E_t = E_{1,2} - E_{3,4}$

$E_t = \therefore 2E_0\cos(\beta d\cos\phi) - 2E_0\cos(\beta d\sin\phi)$

or $\boxed{E_n = \cos(\beta d\cos\phi) - \cos(\beta d\sin\phi)}$

(5)

ملحوظة

1- for any 2 isotropic source

$$E_n = \cos(\psi/2)$$

$\psi = \beta d \cos\theta + \delta \rightarrow$ Phase angle bet. 2 source

- → in phase $\delta = 0$
- → Phase opposition $\delta = 180$
- → Phase quadrature $\delta = 90$
- → " octature $\delta = 45$

1 [illegible]

2- for any n - isotropic source

$$E_n = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

3- for 2 non isotropic source

$$\therefore E_{n\,total} = E_{n\,isotropic} \times E_{individual\ field}$$

ex) short (infinitesimal dipole) $E_n = \sin\theta$
small

$$\therefore E_{n\,total} = \sin\theta * \cos(\psi/2)$$
